'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2023

CC14-PHYSICS

STATISTICAL MECHANICS

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions of the following: 1×5 = 5

   (a) What is the difference between μ-space and Γ-space?

   (b) Distinguish between microstate and macrostate.

   (c) Sketch the Fermi-Dirac distribution function for a gas at $T = 0$ and at $T > 0$.

   (d) State the relation between thermodynamic probability and entropy.

   (e) Explain why does radiation exert pressure.

   (f) What is a Boson? Give examples.

   (g) Can you expect a condensation in electron gas? —Explain.

   (h) How does the lattice vibrational contribution to the specific heat vary with temperature?

GROUP-B

Answer any *three* questions from the following 5×3 = 15

2. (a) What is grand canonical potential? Express grand canonical partition function in terms of the potential. (1+2)+2

   (b) Explain the postulate of 'equal a priori probability'.

3. (a) Using canonical ensemble establish that $\langle E^2 \rangle - \langle E \rangle^2 = k_B T C_V$. 3+2

   (b) Given a Fermi gas, what is the mean occupation number for a state with energy $2k_B T$ above the Fermi energy?

4. (a) Show that for a Fermi-Dirac gas, the average energy per particle is $3E_F/5$ at $T = 0$, where E_F is the Fermi-energy. 3+2

   (b) In BE condensation, show graphically how the condensate fraction varies with temperature.

5. Consider a photon gas enclosed in volume V and in equilibrium at temperature T. Using BE distribution function — 3+1+1

(a) Show that the number of photons having frequency between ν and $\nu + d\nu$ is given by,

$$N(\nu)d\nu = \frac{8\pi r}{c^3}\frac{\nu^2 d\nu}{e^{h\nu/kT}-1}.$$

(b) Hence show that the number of photons in the volume is proportional to T^3.

(c) Using the result in (a) deduce the Planck's law.

6. There are 10 identical particles, each of mass m, to be accommodated in a cubical box of side L. What is the ground-state energy of the system if the particles obey (i) BE-statistics and (ii) FD statistics? 5

GROUP-C

Answer any *two* questions from the following 10×2 = 20

7. (a) Show that for a one-dimensional harmonic oscillator, the fundamental unit of volume of phase space is h, where h is Planck's constant. 5+5

(b) Assuming microcanonical ensemble of a classical ideal gas, deduce the expressions for entropy, internal energy, and specific heat capacity.

8. (a) Consider a system of 2 particles each of which can be in any one of 3 quantum states of respective energies $0, \varepsilon$ and 3ε. The system is in contact with a heat reservoir at temperature T. Calculate the canonical partition function 5+(3+2)

(i) If the particles are distinguishable.

(ii) If the particles are indistinguishable Fermions.

(iii) If the particles are indistinguishable Bosons.

(b) Show that the thermodynamic probability for an ideal gas of N molecules in a volume is of the form $r \propto V^N f(E)$. Hence derive the ideal gas law.

9. (a) A system of identical non-interacting particles of a gas Obey's Pauli's Exclusion Principle. Obtain the distribution law. Discuss (i) the classical limit and (ii) behavior at $T = 0$ K. (5+2) +(1+2)

(b) Define Fermi energy. Calculate the Fermi energy at $T = 0$ K for metallic silver containing one free electron per atom. The density at silver is 10.5 g/c.c. and atomic weight is 108.

10.(a) Derive an expression for electron degeneracy pressure of a white dwarf star. 6+3+1

(b) Use the above result to deduce the mass-radius relation of a white dwarf star.

(c) What is Chandra sekhar mass limit?

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